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Heat and Mass Transfer on the Peristaltic Flow of An Incompressible Electrically Conducting Williamson Fluid through a Porous Medium in a Symmetric Channel with Hall current effects and Inclined Magnetic Field

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Abstract - We studied the peristaltic flow of an incompressible, electrically conducting Williamson fluid in a symmetric planner channel through a porous medium with heat and mass transfer under the influence of inclined magnetic field of an angle of inclination α. Hall effects, viscous dissipation and Joule heating are taken into consideration. The non linear partial differential equations that govern that model were simplified under assumptions of long wavelength and low Reynolds number. Then a regular perturbation technique in the Weissenberg number was applied to obtain a closed form expressions for stream function, axial pressure gradient, temperature and concentration profiles. The influence of various embedded parameters on the flow were plotted through a set of graphs and discussed.

Index Terms - Hall current effects, Heat and mass transfer, MHD flows, Peristaltic transport, porous medium, Williamson fluid
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Nomenclature:

T	Temperature of the fluid,
C	Concentration of the fluid,
T_0	Temperature at $z=0$
T_1	Temperature at $z=h$
C_0	Concentration at $z=0$
C_1	Concentration at $z=h$
V	Velocity vector of the fluid,
J	the current density,
B	the magnetic flux density,
S	the extra stress tensor representing the stresses resulting from a relative motion within the fluid of a Williamson fluid.
K_1	Permeability of the porous medium,
ρ	Density of the fluid,
P	Pressure,
c_p	The specific heat at constant pressure,
σ	Electric conductivity,
a	Mean half width of the channel,
b	Wave amplitude,
λ	Wave length,
c	velocity of propagation
t	Time
T_m	Mean temperature,
k_T	Thermal diffusion ratio,
D	The coefficient of mass diffusivity,
μ	The coefficient of viscosity of the fluid,
ϕ	The viscous dissipation factor
u	the velocity component along the X direction
w	the velocity component along the Z direction
μ_∞	The infinite shear rate viscosity,
μ_0	Zero shear rate viscosity,
Γ	Time constant
Π	The second invariant shear-rate tensor
n_e	The mass of the electron
e	The charge of the electron
β	The hall factor
Re	Reynolds number,
M^2	Hartmann number,
D	Permeability parameter,
Sr	Soret parameter,
Pr	Prandtl number,
Sc	Schimdt number,
Ec	Eckert number,
We	Weissenberg number
Br	Brinkman number

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1 Introduction

The Peristaltic transport is a form of fluid transport via travelling waves imposed on the walls of a distensible fluid, such phenomenon has been documented and mastered through numerous investigations see [1–9]. In physiology, peristaltic mechanism is a neuromuscular property of any smooth muscle structure which transports bio fluids by their propulsive movement, such as transporting urine from kidney to bladder, swallowing of food through esophagus, transport of bile in the bile duct and chyme movement in the intestine. In industrial applications this mechanism can be used in transporting corrosive fluids, sanitary fluids and slurries to avoid contamination with the outer environment. Due to the extensive applications of non-Newtonian fluids in industrial process and in physiological studies, they have gained a considerable attention in the last few decades. Because of the different rheological properties of non-Newtonian fluids, several constitutive equations have been suggested to express such fluids, the computation of such equations presents serious challenges to the researchers in the field, since these equations leads to a set of partial differential equations which are much more non linear and of higher order than the classical Navier–Stokes equations. The Williamson fluid model which is a non-Newtonian fluid that falls into the category of viscoelastic shear thinning fluids, represents the behavior of pseudoplastic materials whose apparent viscosity or consistency decreases instantaneously with increase in rate of shear. Some recent investigations for studying peristaltic flow of non Newtonian fluids are mentioned in the studies [10–17]. The study of magneto hydro dynamic (MHD) peristaltic flow of a fluid is of special interest in certain problems of conductive physiological fluids as the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations for the flow of blood in arteries with arterial disease like arterial stenosis or arteriosclerosis. The usage of the Giant Magneto Resistive (GMR) technology which is a device that applies a magnetic field with a very sensitive sensor, accurately detect the small movements of an object within a magnetic field. This technology has the potential to facilitate the study of peristaltic activity in some tubular structures such as a bowel, fallopian tube and perhaps even in the vas deferens. Hayat et al. [17] have analyzed the peristaltic transport of a Jeffrey fluid under effects of an endoscope and applied magnetic field. Mekheimer [18] has examined the peristaltic transport of a couple stress fluid under influence of an induced magnetic field. Abo-Eldahab et al. [19] have studied the effects of Hall currents on peristaltic transport of a couple stress fluid. Furthermore, the effects of porous medium on peristalsis is useful in studying some biomedical applications like transport process in lungs, kidneys, gallbladder with stones. The first attempt to study the peristaltic flow through a porous medium was presented by Elsehawy [20]. Elsehawy et al. [21] have studied the peristaltic motion of generalized Newtonian fluid through a porous medium. Hayat et al. [22] have investigated the Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Abdelmaboud and Mekheimer [23] have discussed the peristaltic transport of a second order fluid through a porous medium. Dharmenda [24] has investigated the peristaltic hemodynamic flow of a couple-stress fluid through a porous medium with slip effects. Few attempts have been devoted to peristaltic flows of non Newtonian fluids in presence of heat and mass transfer; such investigations are of great importance, which is due to their extensive applications in medical and bio-engineering sciences, as it may be relevant in many processes in human body, like oxygenation in lungs, hemodialysis and nutrients diffuse out of blood. Nadeem and Akbar [25] have studied the influence of heat and mass transfer on the peristaltic flow of Johnson Segalman fluid in a vertical asymmetric channel with induced magnetic field, and in an earlier study they have discussed the peristaltic flow of radial varying magnetic field in an annulus with heat and mass transfer. Hayat and Hania [26] have investigated the effects of heat and mass transfer on peristaltic flow of Williamson fluid in a non uniform channel with slip conditions. Nadeem et al. [27] have discussed the influence of heat and mass transfer on the peristaltic flow of a third order fluid in a diverging tube. Eldabe and Abu-Zied [28] have investigated the wall properties effect on peristaltic transport of micropolar fluid in presence of heat and mass transfer. Nadeem.S and Safia Akram [30] have presented a peristaltic flow of a Williamson model in an asymmetric channel. The governing equations of Williamson model in two dimensional peristaltic flow phenomena are constructed under long wave length and low Reynolds number approximations. For large Williamson parameter *We*, the curves of the pressure rise are not linear but for very small *We* it behave like a Newtonian fluid. Abbasi Fahad Munir, Hayat Tasawar and Ahmad Bashir [31] discussed the peristaltic transport of viscous fluid in an asymmetric channel where the channel walls exhibit convective boundary conditions and consider the joule heating. Awais.M et al. [32] investigated Magneto hydro dynamic peristaltic flow of Jeffery fluid in an asymmetric channel where the channel walls satisfy the convective conditions. Hayat and Abbasi [33] gave reports on the effects of velocity and thermal slip parameters on the peristaltic motion of variable viscosity and magneto hydro dynamic (MHD) fluid in an asymmetric channel. The mathematical model describing the slip peristaltic flow of nano fluid was analytically investigated by Abdelhalim Ebaid and Emad H. Aly [34]. Safia Akram et al. [35] investigated the peristaltic flow of a Maxwell fluid in a porous asymmetric channel through a porous medium. Despite all such challenges, various recent researchers are even making their valuable contributions for peristaltic transport of non-Newtonian fluids [36-50]. Motivated by the facts discussed above, the aim of the present work is to investigate the hall current effects on the peristaltic flow of an incompressible electrically conducting Williamson fluid through a porous medium in presence of heat and mass transfer. In addition, viscous dissipation, Joule heating and Soret effects are taken into consideration. We introduce the basic equations that govern the model. We obtain the solution of the problem using the regular perturbation technique in terms of small Weissenberg number. Discussion of results and conclusion was made through a set of plots.

2 Formulation and Solution of the Problem

We consider the peristaltic transport of an incompressible, electrically conducting Williamson fluid in a symmetric planner channel through a porous medium with heat and mass transfer and under the influence of inclined magnetic field of an angle of inclination α as shown in the Figure 1. Hall effects, viscous dissipation and Joule heating are taken into consideration. The basic equations that govern the flow of MHD incompressible non-Newtonian fluid through a porous medium in presence of heat and mass transfer with the effects of viscous dissipation, Joule heating and thermo diffusion are, The continuity equation

$$\nabla . V = 0 \tag{1}$$

Momentum equation

$$\rho \frac{dV}{dz} = -\nabla P + \nabla \cdot S + J \times B - \frac{\mu}{K_1} V \tag{2}$$

The heat equation

$$\rho c_p \frac{dT}{dz} = k\nabla^2 T + \mu\phi + \frac{1}{\sigma} J \cdot J \tag{3}$$

The concentration equation

$$\frac{dC}{dz} = D\nabla^2 C + \frac{Dk_T}{T_m}\nabla^2 T \tag{4}$$

The constitutive equation for the extra stress tensor S is [29],

$$S = \left(\mu_\infty + (\mu_0 - \mu_\infty)(1 - \Gamma\dot{\gamma})^{-1}\right)\dot{\gamma} \tag{5}$$

and $\dot{\gamma}$ is defined by

$$\dot{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \dot{\gamma}_{ij}\dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\Pi} \tag{6}$$

By considering $\mu_\infty = 0$ and $\Gamma\dot{\gamma}$ in the constitutive equation (5), so we can write

$$S = -\mu_0\left[(1 + \Gamma\dot{\gamma})\right]\dot{\gamma} \tag{7}$$

In which Eq. (7) reduces to a Newtonian fluid in case $\Gamma = 0$.

We consider the peristaltic flow of an incompressible, electrically conducting Williamson fluid through a porous medium in a three dimensional symmetric flexible channel of width 2a taking hall current into account. The flow is considered in the direction of X-axis and Z-axis is taken normal to the flow. A sinusoidal wave of amplitude b propagates along the channel walls with constant speed c along the direction of the X-axis. A strong uniform magnetic field with magnetic flux density $B = (0, B_0 \sin\alpha, 0)$ is applied normal to the channel. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number $(\mathrm{Re}_m \ll 1)$, also it is assumed that there is no applied or polarization voltage so that the total electric field $E = 0$. The geometry of the wall surface is described by

$$H(X,t) = \pm a \pm b\cos\left(\frac{2\pi}{\lambda}(X - ct)\right) \tag{8}$$

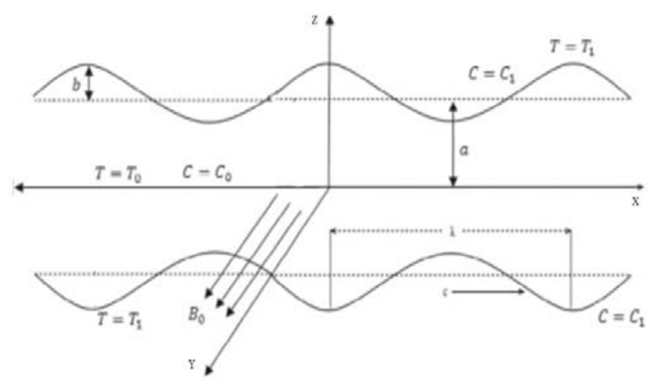

Fig. 1: Physical configuration of the problem

The generalized Ohm's law can be written as

$$J = \sigma\,(E + V \times B - \beta\, J \times B) \tag{9}$$

Where $\beta = 1/n_e e$ is the hall factor,

Eq. (9) can be solved in J to yield the Lorentz force vector in the form

$$J \times B = -\frac{\sigma B_0 \sin\alpha}{1 + m^2 \sin^2\alpha}\left[(Um\sin\alpha - W)\bar{i} + (U + Wm\sin\alpha)\bar{k}\right] \tag{10}$$

Where, U and W are the X and Z components of the velocity vector, and $m = \sigma B_0 \beta$ is hall parameter.

Using the transformations

$$x = X - ct,\ y = Y,\ u = U - c,\ w = W,\ p(x) = P(X,t) \tag{11}$$

The unsteady flow in the fixed frame (X, Z) appears steady in the wave frame (x, z) in which are the velocity components in the wave frame.

Introducing non-dimensional the following non-dimensional quantities

$$X^* = \frac{X}{\lambda},\ Z^* = \frac{Z}{\lambda},\ u^* = \frac{u}{c},\ w^* = \frac{w}{c},$$

$$t^* = \frac{c}{\lambda}t,\ \dot{\gamma}^* = \dot{\gamma}\frac{a}{c},\ p^* = \frac{a^2 p}{\mu c \lambda},$$

$$\psi^* = \frac{\psi}{ca},\ h = \frac{H}{a},\ \theta^* = \frac{T - T_0}{T_1 - T_0},\ C^* = \frac{C - C_0}{C_1 - C_0} \tag{12}$$

Making use of non-dimensional variables, the equations (2)-(4) reduces with respected to the wave frame, the governing equations in terms of the stream function ψ are (dropping asterisks),

where $u^* = \dfrac{\partial\psi}{\partial z}$ and $w^* = -\delta\dfrac{\partial\psi}{\partial x}$.

$$\delta \operatorname{Re}\left[\left(\frac{\partial\psi}{\partial z}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial z}\right)\frac{\partial\psi}{\partial z}\right]=-\frac{\partial p}{\partial x}+2\delta^2\frac{\partial}{\partial x}\left((1+We\dot{\gamma})\frac{\partial^2\psi}{\partial x\partial z}\right)+\frac{\partial}{\partial z}\left((1+We\dot{\gamma})\left(\frac{\partial^2\psi}{\partial z^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)\right)+\delta\left[\frac{mM^2\sin^3\alpha}{1+m^2\sin^2\alpha}\right]\frac{\partial\psi}{\partial x}-\left[\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{D}\right]\left(\frac{\partial\psi}{\partial z}+1\right) \quad (13)$$

$$-\delta^3 \operatorname{Re}\left[\left(\frac{\partial\psi}{\partial z}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial z}\right)\frac{\partial\psi}{\partial x}\right]=-\frac{\partial p}{\partial z}+\delta^2\frac{\partial}{\partial x}\left((1+We\dot{\gamma})\left(\frac{\partial^2\psi}{\partial z^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)\right)+-2\delta^2\frac{\partial}{\partial z}\left((1+We\dot{\gamma})\frac{\partial^2\psi}{\partial x\partial z}\right)-\delta\left[\frac{mM^2\sin^3\alpha}{1+m^2\sin^2\alpha}\right]\left(\frac{\partial\psi}{\partial z}+1\right)-\delta^2\left[\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{\delta D}\right]\frac{\partial\psi}{\partial x} \quad (14)$$

$$\delta \operatorname{Re}\left(\frac{\partial\psi}{\partial z}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial z}\right)=\frac{1}{\Pr}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}\right)+Ec(1+We\dot{\gamma})\left(4\delta^2\left(\frac{\partial^2\psi}{\partial x\partial z}\right)^2+\left(\frac{\partial^2\psi}{\partial z^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2\right)+Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left[\delta^2\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial z}+1\right)^2\right] \quad (15)$$

$$\delta \operatorname{Re}\left(\frac{\partial\psi}{\partial z}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial z}\right)=\frac{1}{Sc}\left(\delta^2\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial z^2}\right)+Sr\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial z^2}\right) \quad (16)$$

Where,

$\dot{\gamma}=\left(4\delta^2\left(\frac{\partial^2\psi}{\partial x\partial z}\right)^2+\left(\frac{\partial^2\psi}{\partial z^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2\right)^{1/2}$, $\operatorname{Re}=\frac{\rho c d_1}{\mu}$ is the Reynolds number, $M^2=\frac{\sigma B_0^2 a}{\mu^2}$ is the Hartmann number, $D=\frac{K_1}{a^2}$ is the permeability parameter, $Sr=\frac{\rho D k(T_1-T_0)}{T_m\mu(C_1-C_0)}$ is the Soret parameter, $\Pr=\frac{\rho\nu\xi}{k}$ is the Prandtl number, $Sc=\frac{\mu}{\rho D}$ is the Schimdt number, $Ec=\frac{c^2}{\xi(T_1-T_0)}$ is the Eckert number, $We=\frac{\Gamma c}{a}$ is the Weissenberg number and $Br=\Pr Ec$ is the Brinkman number.

And under the assumptions of long wavelength $\delta \ll 1$ and low Reynolds number, we obtain

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left(\left(1+We\frac{\partial^2\psi}{\partial z^2}\right)\frac{\partial^2\psi}{\partial z^2}\right)-\left[\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{D}\right]\left(\frac{\partial\psi}{\partial z}+1\right) \quad (17)$$

$$0=-\frac{\partial p}{\partial z} \quad (18)$$

$$0=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial z^2}+Ec\left(\left(\frac{\partial^2\psi}{\partial z^2}\right)^2+We\left(\frac{\partial^2\psi}{\partial z^2}\right)^3\right)+Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{\partial\psi}{\partial z}+1\right)^2 \quad (19)$$

$$0=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2}+Sr\frac{\partial^2\theta}{\partial z^2}. \quad (20)$$

Eq. (18) implies that $p \neq p(z)$. And we can write the equation (17) in the form

$$0=\frac{\partial^2}{\partial z^2}\left(\left(1+We\frac{\partial^2\psi}{\partial z^2}\right)\frac{\partial^2\psi}{\partial z^2}-\left(\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{D}\right)\psi\right) \quad (21)$$

The non-dimensional boundary conditions in the wave frame are given as

$$\psi = 0, \frac{\partial^2 \psi}{\partial z^2} = 0, \theta = 0, C = 0 \quad \text{at} \quad z = 0 \tag{22}$$

$$\psi = q, \frac{\partial \psi}{\partial z} = -1, \theta = 1, C = 1 \quad \text{at} \quad z = h \tag{23}$$

Where, $h(x) = 1 + \varepsilon\cos(2\pi x)$, $\varepsilon = b/a (0 < \varepsilon < 1)$ is the amplitude ratio and q is the dimensionless time mean flow rate in the wave frame. It is related to the dimensionless time mean flow rate Q in the laboratory frame through the relation $Q = q + 1$.

The non dimensional expression of pressure rise ΔP per wave length is

$$\Delta P = \int_0^1 \frac{dp}{dx} dx \tag{24}$$

Solving the equations (19), (20) and (21) with the boundary conditions using a regular perturbation technique in terms of the small parameter as,

$$\psi = \psi_0 + We\psi_1 + We^2\psi_2 + \cdots\cdots\cdots \tag{25}$$

$$q = q_0 + We\, q_1 + We^2 q_2 + \cdots\cdots\cdots \tag{26}$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + We\frac{dp_1}{dx} + We^2\frac{dp_2}{dx} + \cdots\cdots\cdots \tag{27}$$

$$\theta = \theta_0 + We\theta_1 + We^2\theta_2 + \cdots\cdots\cdots \tag{28}$$

$$C = C_0 + We\, C_1 + We^2 C_2 + \cdots\cdots\cdots \tag{29}$$

Substituting the equations (25) - (29) into equations (19) - (21) and then comparing the coefficients of like powers of *We* up to the first order and neglecting powers of order 2 and higher, we obtain

Zeroth order:

$$0 = -\left[\frac{\partial^2}{\partial z^2} - \left(\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha} + \frac{1}{D}\right)\right]\frac{\partial^2\psi_0}{\partial z^2} \tag{30}$$

$$0 = \frac{1}{\Pr}\frac{\partial^2\theta_0}{\partial z^2} + Ec\left(\frac{\partial^2\psi}{\partial z^2}\right)^2 + Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{\partial\psi_0}{\partial z} + 1\right)^2 \tag{31}$$

$$0 = \frac{1}{Sc}\frac{\partial^2 C_0}{\partial z^2} + Sr\frac{\partial^2\theta_0}{\partial z^2} \tag{32}$$

Corresponding boundary conditions are

$$\psi_0 = 0, \frac{\partial^2\psi_0}{\partial z^2} = 0, \theta_0 = 0, C_0 = 0 \quad \text{at} \quad z = 0 \tag{33}$$

$$\psi_0 = q_0, \frac{\partial\psi_0}{\partial z} = -1, \theta_0 = 1, C_0 = 1 \quad \text{at} \quad z = h \tag{34}$$

First order:

$$0 = \frac{\partial^4\psi_1}{\partial z^4} - \left(\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha} + \frac{1}{D}\right)\frac{\partial^2\psi_1}{\partial z^2} + \frac{\partial^2}{\partial z^2}\left(\frac{\partial^2\psi_1}{\partial z^2}\right)^2 \tag{35}$$

$$0 = \frac{1}{\Pr}\frac{\partial^2\theta_1}{\partial z^2} + Ec\left(2\frac{\partial^2\psi_0}{\partial z^2}\frac{\partial^2\psi_1}{\partial z^2} + \left(\frac{\partial^2\psi_0}{\partial z^2}\right)^3\right) + Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left[2\left(\frac{\partial\psi_0}{\partial z} + 1\right)\frac{\partial\psi_1}{\partial z}\right] \tag{36}$$

$$0 = \frac{1}{Sc}\frac{\partial^2 C_1}{\partial z^2} + Sr\frac{\partial^2\theta_1}{\partial z^2} \tag{37}$$

Corresponding boundary conditions are

$$\psi_1 = 0, \frac{\partial^2\psi_1}{\partial z^2} = 0, \theta_1 = 0, C_1 = 0 \quad \text{at} \quad z = 0 \tag{38}$$

$$\psi_1 = q_1, \frac{\partial\psi_1}{\partial z} = 0, \theta_1 = 0, C_1 = 0 \quad \text{at} \quad z = h \tag{39}$$

Solving the above resulting zeroth and first order equations using the relavent boundary conditions, we obtained the exact forms for the stream function ψ, the longitudinal velocity $\partial\psi/\partial z$, the pressure gradient dp/dx, the temperature distribution θ and the concentration distribution C. The expressions are mentioned in the appendix.

4 Results and Discussion

We have presented a set of Figures (2–9), that describe qualitatively the effects of various parameters of interest on flow quantities such as the axial velocity *u*, pressure rise per wave length ΔP, axial pressure gradient dp/dx, temperature distribution θ and concentration distribution C.

Figures 2(a-e) display the variation of axial pressure gradient dp/dx with *x* for different values of permeability parameter *D*, Hartman number *M*, an angle of inclination α, hall parameter *m* and the Weissenberg number *We*. The following results can be observed from these figures. The magnitude of the pressure gradient decreases with the increase in *D*, *m*, *We* and α, while it increases with the increase in *M*. It is also observed that the maximum pressure gradient occurs at the narrow part of the channel. The pressure rise per wavelength ΔP against flow rate *Q* for different values of *D*, *m*, *M*, α and *We* are shown in Figures 3 (a-d). It is observed from these figures that, in the pumping region $(\Delta P > 0)$ the pumping rate decreases by increasing *D*, *m* and *We*, while in the co-pumping region $(\Delta P < 0)$, the pumping rate decreases by increasing *M* or α and increases by increasing *D*, *m* and *We*. For the free pumping case $(\Delta P = 0)$, there are no noticeable differences observed. The variations of temperature distribution θ with for several values of *M*, α, *We*, *m* and the Brinkman number *Br* are plotted in Figures 4 (a-e). These figures depict an increase in the temperature field when *M* or α and *Br* increases and a decrease in the temperature field when *We* and *m* increases. It is clear from the last term in equation (19) that an increase in the hall parameter

m will result in a decrease of the Joule dissipation which is proportional to $1/1+m^2$ and hence a decrease in the temperature distribution. Whereas an increase in the Brinkman number *Br* means a more energy is stored in the fluid due to the frictional forces and thus an increase in the temperature distribution. Figures 5 (a-f) represent the concentration distribution C for different variations of *M*, α, *m*, *We*, the Soret number *Sr* and the amplitude ratio ε. It can be noticed from these figures that the concentration distribution decreases by increasing *M*, α, *Sr* and ε, while it increases when *m* and *We* increases. Figures 6 (a-f) are prepared to study the role of *M*, α, *m*, *We*, ε and *D* on the axial velocity *u*. It is obvious from Figures 6(a), 6(e) and 6(f) that an increase in *m*, *D* and ε, the magnitude of the velocity increases at the center of the channel whereas it decreases near the channel walls. From Figures 6(b), 6(c) and 6(d) it is observed that *M*, α and *We* has an opposite behaviour to that of *m* and *We*. The opposite effects of *m* and *M* on the longitudinal velocity *u* is quite opposite in accordance with physical argument, since the effective conductivity σ of the fluid is decreased by increasing *m*; resulting in less resistivity of the Lorentz force and therefore an increase in the fluid velocity at the center of the channel, While increasing *M* results in an increase in the damping force that will decrease the fluid velocity at the center of the channel. We would like to refer to a conclusion we have reached upon resolving our problem after neglecting the Joule heating effect in the heat equation and then examining the effects of the Hartman number *M* and the hall parameter *m* on the temperature and the concentration distributions through plots. From Figures 7 (a-d), we found out from Figures 7(a) and 7(b), that the temperature distribution decreases by an increase in *M* and increases by an increase in *m*. Figures 7(c) and 7(d), show that the concentration distribution decreases by an increase in m and increases by an increase in *M*. Therefore, in our present model and from the above discussion it is clear that, when neglecting the Joule heating term in the heat equation, the roles of *M* and *m* on the temperature and concentration distributions are reversed. Figures (8-9) represents the behaviour of streamlines for the different values of *We* and α. Figures 8 (a) and 8 (b) examine that size of trapped bolus decreases when *We* increases. Figures 9 (a) and 9 (b) shown that the size of trapped bolus increases with an increase in an angle of an inclination α.

4 Conclusion

We studied the Hall effects on peristaltic transport of a Williamson fluid in a symmetric channel through a porous medium with heat and mass transfer and under the influence of inclined magnetic field of an angle of inclination α, viscous dissipation, Joule heating and thermo diffusion effects are taken into consideration. The governing three dimensional equations have been simplified under the assumptions of low Reynolds number and long wavelength. The simplified equations are solved analytically using regular perturbation technique. The main observations have been pointed out as follows. (1). The axial pressure gradient decreases with an increase in *m*, *D* and α while it decreases with an increase in *M* and *We*. (2). The temperature field increases with an increase in *M* and α. It decreases with an increase in *m* and *We*. (3). The concentration field decreases with an increase in *M* or α. It increases with an increase in *m* and *We*. (4). The effect of *M* or α and *We* on the longitudinal velocity are quite opposite to that of *m*, ε and *D*. (5). In the absence of the Joule heating effect the roles of *M*, α and *m* on the temperature and concentration distributions are reversed. So it is important not to neglect the Joule heating effect in the temperature equation in order to obtain more accurate results. (6). The size of trapped bolus decreases when *We* increases where as it increases with an increase in α.

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GRAPHS

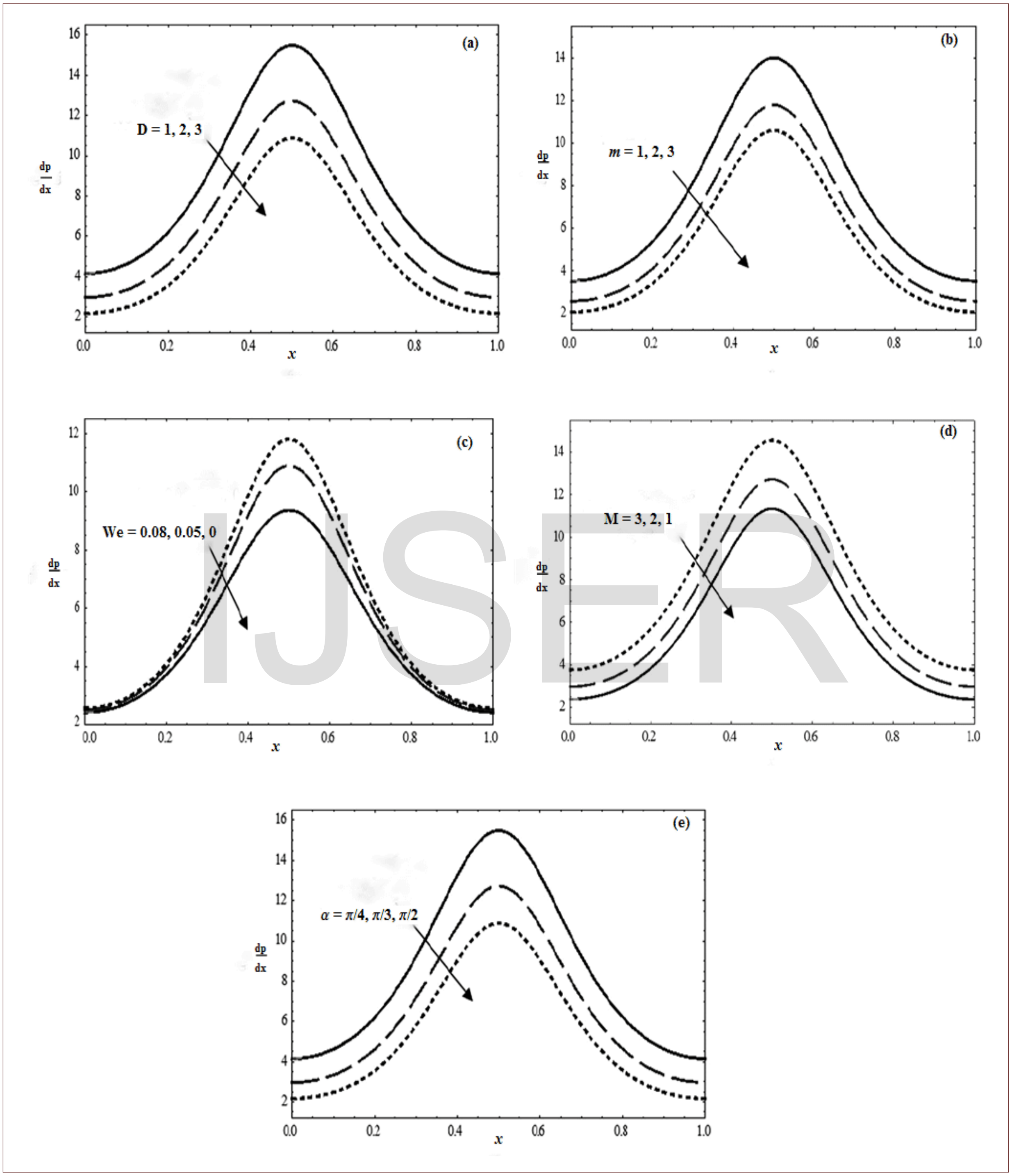

Fig. 2: The variation of pressure gradient dp/dx against D, m, We, M and α with $\varepsilon = 0.2, Q = -1$

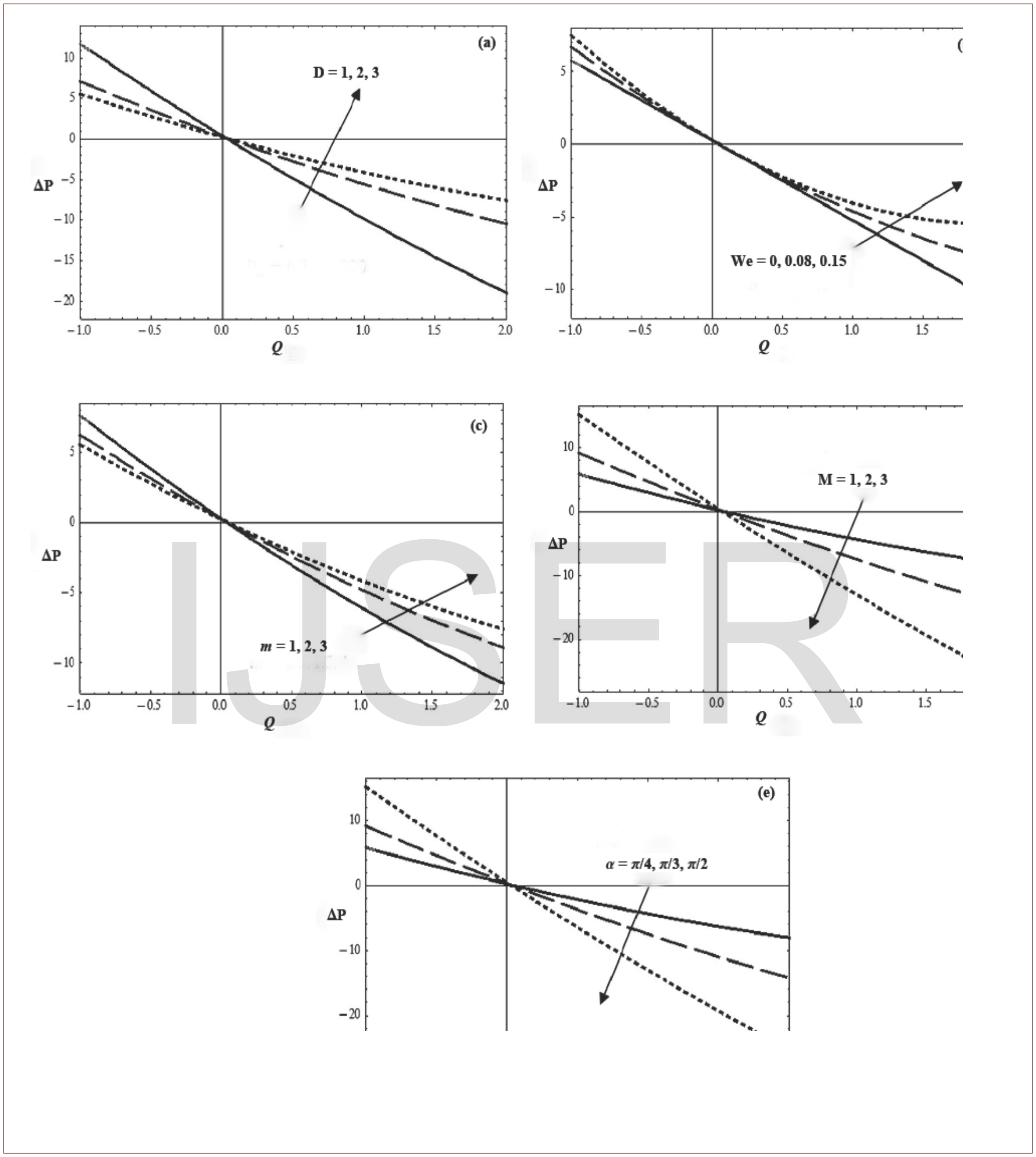

Fig. 3: The variation of Pressure rise ΔP against D, We, m, M and α with $\varepsilon = 0.2$

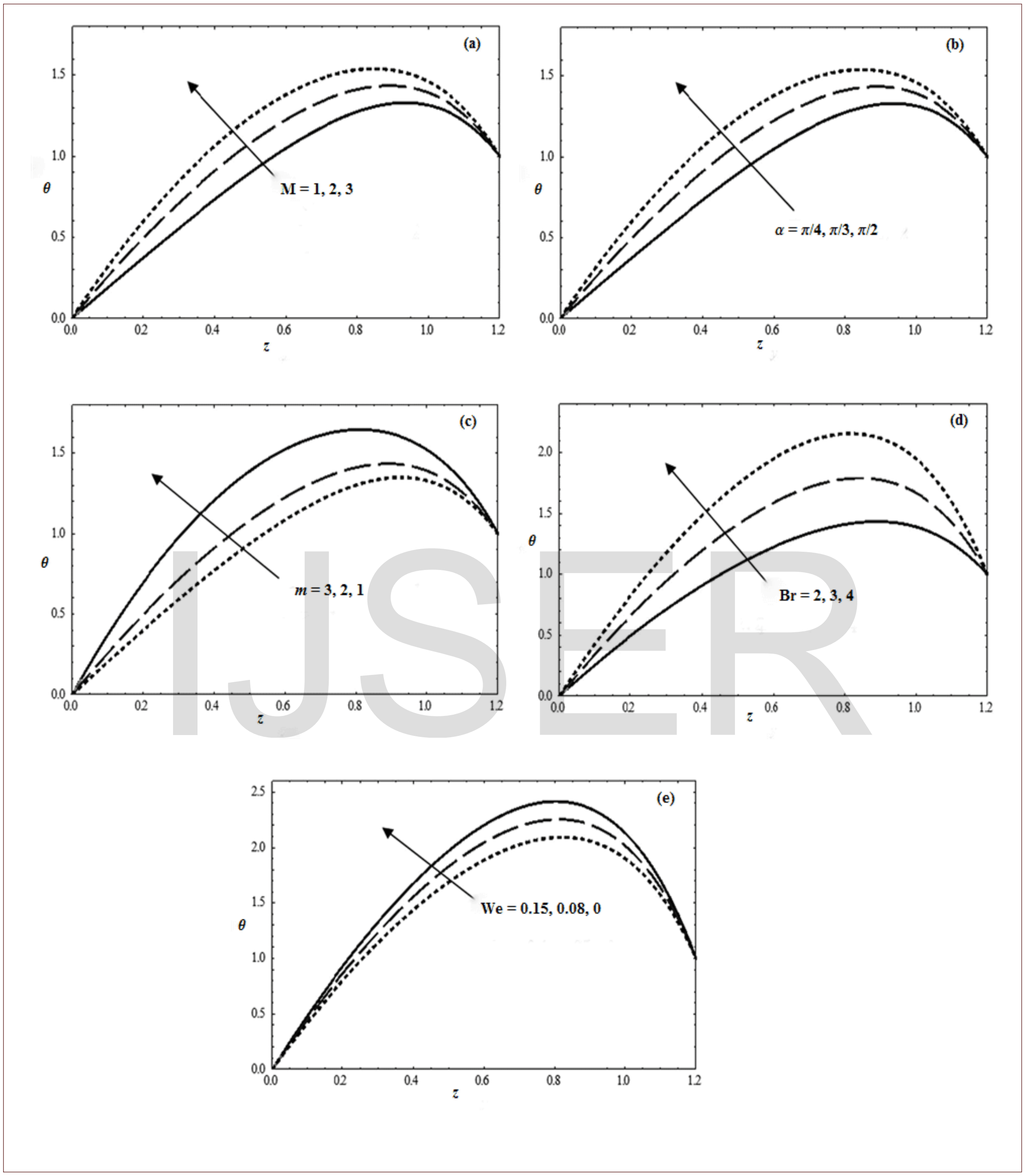

Fig. 4: Temperature distribution θ with M, α, m, Br and We with $D=0.8, \varepsilon=0.2, q_0=0.05, q_1=0.05, x=0$

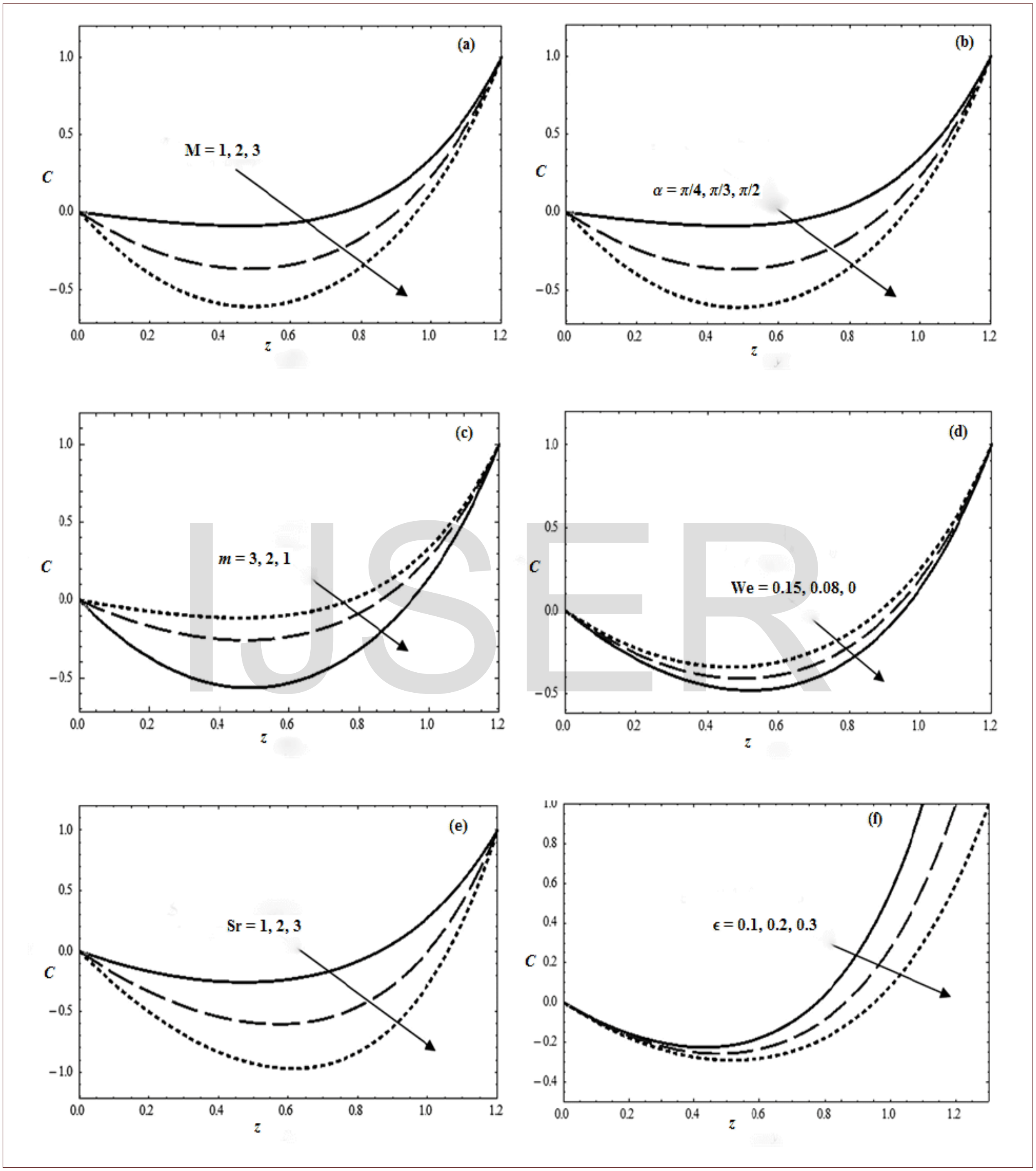

Fig. 5: Concentration C with M, α, m, We, Sr and ε with $D = 0.8$, $Br = 2$, $q_0 = 0.05$, $q_1 = 0.05$, $x = 0$

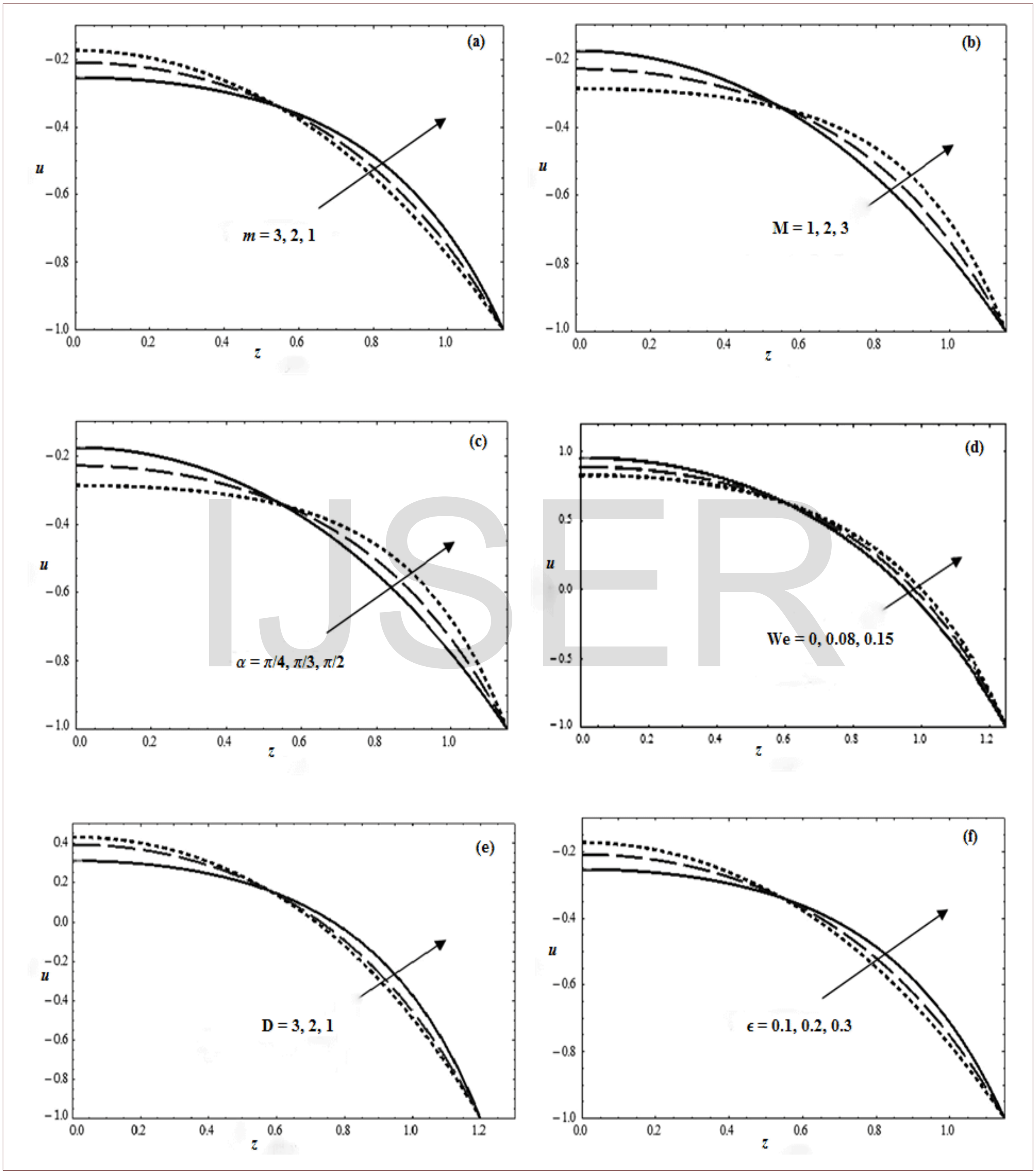

Fig. 6. The velocity profiles for m, M, α, We and D with
$\varepsilon = 0.2, Q = 1.5, Br = 2, We = 0.08, q_0 = 0.05, q_1 = 0.05, x = 0, Sc = 1.5, Sr = 1.5$

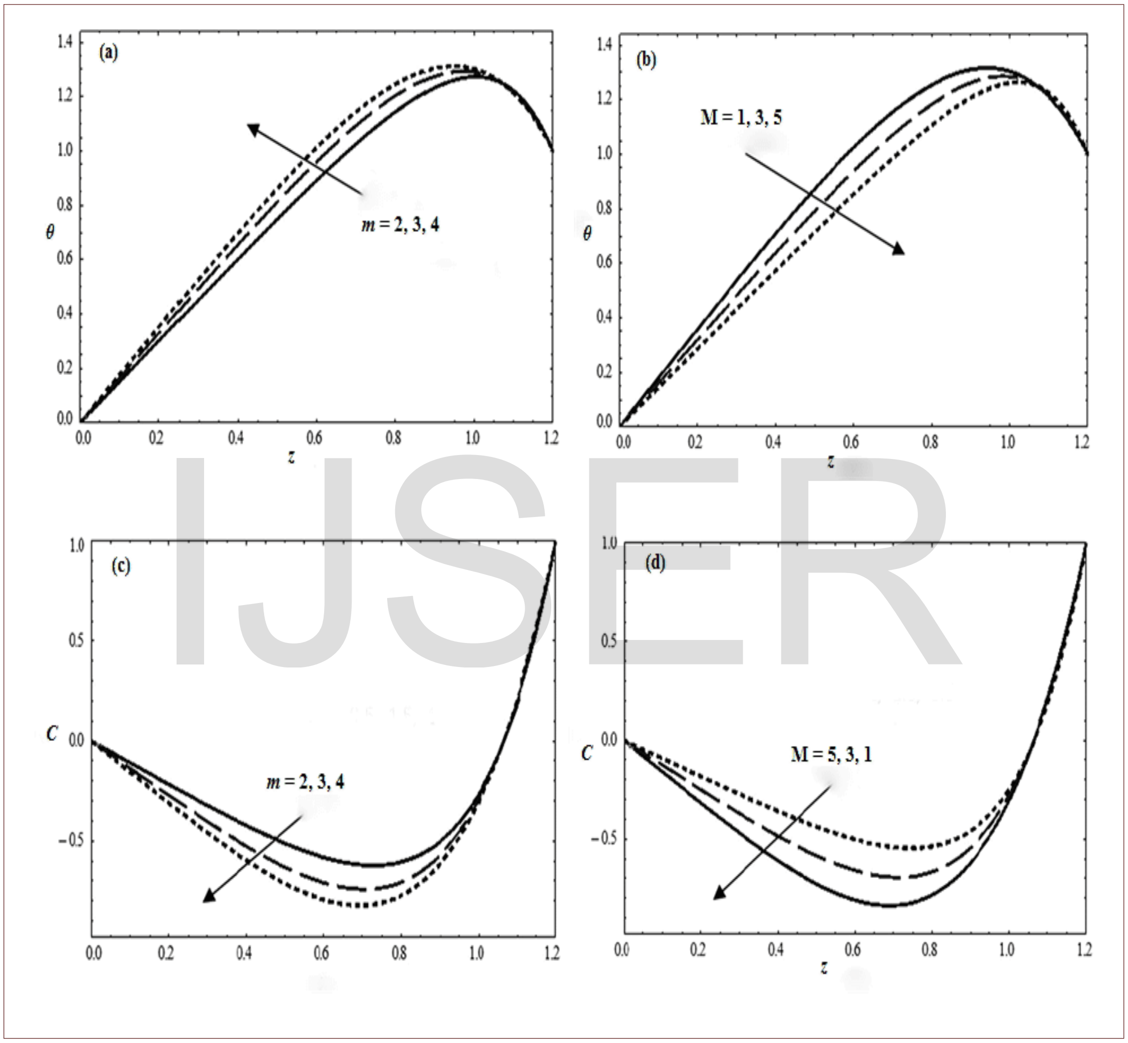

Fig. 7: The temperature and Concentration with m and M with
$D = 0.8,\ Br = 2,\ \varepsilon = 0.2,\ We = 0.08,\ q_0 = 0.05,\ q_1 = 0.05,\ Sr = 1.5, Sc = 1.5, x = 0$

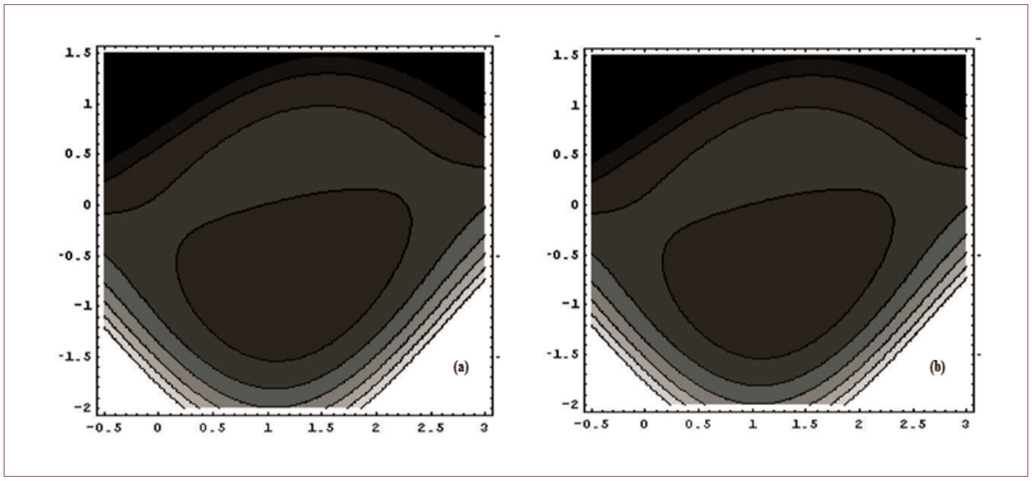

Fig. 8. The stream lines (a). $We = 0.05$ (b). $We = 0.08$
$\varepsilon = 0.2, Q = 1.5, M = 2, Br = 2, D = 0.8, q_0 = 0.05, q_1 = 0.05, x = 0, Sc = 1.5, Sr = 1.5, \alpha = \pi / 4$

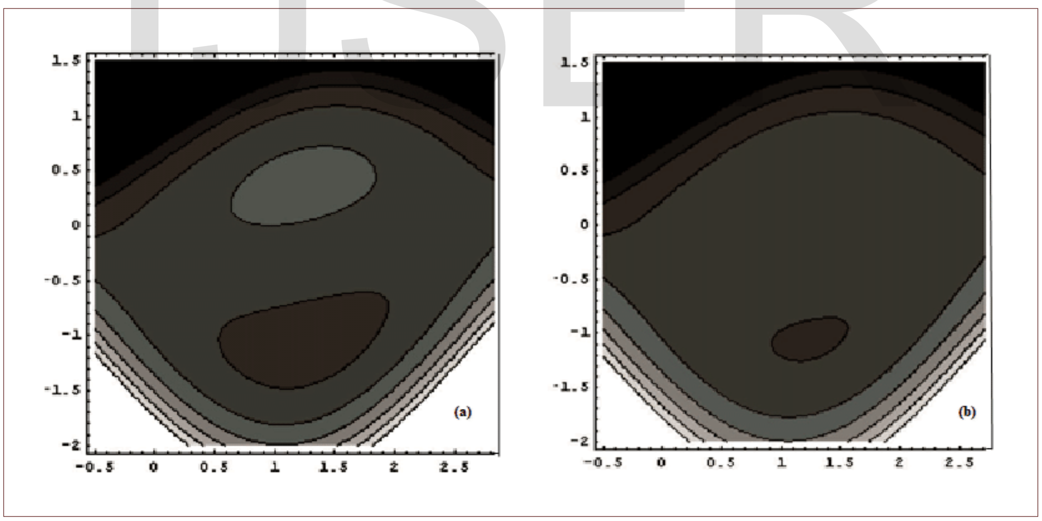

Fig. 9. The stream lines (a). $\alpha = \pi / 4$ (b). $\alpha = \pi / 3$
$\varepsilon = 0.2, Q = 1.5, Br = 2, M = 2, D = 0.8, q_0 = 0.05, q_1 = 0.05, x = 0, Sc = 1.5, Sr = 1.5, We = 0.05$

APPENDIX:

$$\psi=\psi_0+We\,\psi_1\,,\ \frac{\partial\psi}{\partial z}=C_2+\lambda C_3e^{\lambda z}-\lambda C_4e^{-\lambda z}+We\left(B_2+\lambda\left(B_3e^{\lambda z}-B_4e^{-\lambda z}\right)+\frac{2\lambda^3}{1-4\lambda}\left(C_3^2e^{2\lambda z}-C_4^2e^{-2\lambda z}\right)\right)$$

$$\psi_0=C_2z+C_3e^{\lambda z}+C_4e^{-\lambda z}\ ,\ \theta_0=-\Pr Ec(C_3e^{\lambda z}+C_4e^{-\lambda z})-\Pr Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{z^2}{2}+C_2\frac{z^2}{2}+C_3\frac{e^{\lambda z}}{\lambda}-C_4\frac{e^{-\lambda z}}{\lambda}\right)+A_1z+A_2$$

$$C_0=Sr\,Sc\left\{\Pr Ec(C_3e^{\lambda z}+C_4e^{-\lambda z})+\Pr Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{z^2}{2}+C_2\frac{z^2}{2}+C_3\frac{e^{\lambda z}}{\lambda}-C_4\frac{e^{-\lambda z}}{\lambda}\right)\right\}+A_3z+A_4$$

$$\psi_1=B_1+B_2z+B_3e^{\lambda z}+B_4e^{-\lambda z}+\frac{\lambda^3}{1-4\lambda}\left(C_3^2e^{2\lambda z}+C_4^2e^{-2\lambda z}\right)$$

$$\theta_1=-\Pr Ec\left\{2\lambda^2\left(\frac{a_{12}}{4\lambda^2}e^{2\lambda z}+a_{13}\frac{z^2}{2}+\frac{a_{14}}{4\lambda^2}e^{-2\lambda z}+a_{15}\left[\frac{a_{16}}{9\lambda^2}e^{3\lambda z}+\frac{a_{17}}{\lambda^2}e^{-\lambda z}+\frac{a_{18}}{\lambda^2}e^{\lambda z}+\frac{a_{19}}{9\lambda^2}e^{-3\lambda z}\right]\right)\right\}$$

$$-\Pr Ec\frac{2M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{a_{22}z^2}{2}+\frac{a_{23}}{\lambda}e^{\lambda z}+\frac{a_{24}}{\lambda}e^{-\lambda z}+\frac{a_{25}}{4\lambda^2}e^{2\lambda z}+\frac{a_{26}}{4\lambda^2}e^{-2\lambda z}+\frac{a_{27}}{9\lambda^2}e^{3\lambda z}+\frac{a_{28}}{9\lambda^2}e^{-3\lambda z}\right)+D_1z+D_2$$

$$C_1=SrSc\left\{\Pr Ec\left[2\lambda^2\left(\frac{a_{12}e^{2\lambda z}}{4\lambda^2}+a_{13}\frac{z^2}{2}+\frac{a_{14}e^{-2\lambda z}}{4\lambda^2}+\right.\right.\right.$$

$$\left.\left.a_{15}\left[\frac{a_{16}e^{3\lambda z}}{9\lambda^2}+\frac{a_{17}e^{-\lambda z}}{\lambda^2}+\frac{a_{18}e^{\lambda z}}{\lambda^2}+\frac{a_{19}e^{-3\lambda z}}{9\lambda^2}\right]\right)+\lambda^6\left[\frac{a_{16}e^{3\lambda z}}{9\lambda^2}+\frac{a_{20}e^{\lambda z}}{\lambda^2}+\frac{a_{21}e^{-\lambda z}}{\lambda^2}+\frac{a_{19}e^{-3\lambda z}}{9\lambda^2}\right]\right]+$$

$$\left.+\Pr Ec\frac{2M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{a_{22}z^2}{2}+\frac{a_{23}e^{\lambda z}+a_{24}e^{-\lambda z}}{\lambda}+\frac{a_{25}e^{2\lambda z}+a_{26}e^{-2\lambda z}}{4\lambda^2}+\frac{a_{27}e^{3\lambda z}+a_{28}e^{-3\lambda z}}{9\lambda^2}\right)\right\}+F_1z+F_2$$

$$\frac{dp}{dx}=2we^2\lambda^5\left[B_3e^{\lambda z}-B_4e^{-\lambda z}+\frac{32\lambda^3}{1-4\lambda}\left(C_3^2e^{2\lambda z}-C_4^2e^{-2\lambda z}\right)\right]+$$

$$+we\lambda^3\left[B_3e^{\lambda z}-B_4e^{-\lambda z}+\frac{8\lambda^3}{1-4\lambda}\left(C_3^2e^{2\lambda z}-C_4^2e^{-2\lambda z}\right)\right]+$$

$$-\left(\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{D}\right)\left[1+we\left(B_2+\lambda B_3e^{\lambda z}-\lambda B_4e^{-\lambda z}+\frac{2\lambda^4}{1-4\lambda}\left(C_3^2e^{2\lambda z}-C_4^2e^{-2\lambda z}\right)\right)\right]+$$

$$+2We\lambda^5\left(C_3e^{\lambda z}+C_4e^{-\lambda z}\right)+\lambda^3\left(C_3e^{\lambda z}-C_4e^{-\lambda z}\right)-\left(\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{D}\right)\left(1+C_2+C_3e^{\lambda z}+C_4e^{-\lambda z}\right)$$

$$\lambda=\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}+\frac{1}{D}\,,\ C_2=-1-\frac{\lambda(e^{\lambda h}+e^{-\lambda h})(q_0+h)}{(e^{\lambda h}-e^{-\lambda h})-\lambda h(e^{\lambda h}+e^{-\lambda h})}$$

$$C_3=\frac{q_0+h}{(e^{\lambda h}-e^{-\lambda h})-\lambda h(e^{\lambda h}+e^{-\lambda h})}\,,\ C_4=\frac{-(q_0+h)}{(e^{\lambda h}-e^{-\lambda h})-\lambda h(e^{\lambda h}+e^{-\lambda h})}\,,$$

$$A_1 = \frac{1}{h}\left\{1 + \Pr Ec(C_3 e^{\lambda h} + C_4 e^{-\lambda h}) + \Pr Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{h^2}{2} + C_2\frac{h^2}{2} + C_3\frac{e^{\lambda h}}{\lambda} + C_4\frac{e^{-\lambda h}}{\lambda}\right)\right.$$

$$\left. -\Pr Ec(C_3+C_4) + \frac{\Pr Ec}{\lambda}\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(C_3 - C_4\right)\right\}$$

$$A_2 = \Pr Ec(C_3+C_4) + \frac{\Pr Ec}{\lambda}\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(C_3 - C_4\right)$$

$$A_3 = \frac{1}{h}\left\{1 - Sr\,Sc\left(\Pr Ec(C_3 e^{\lambda h} + C_4 e^{-\lambda h}) + \Pr Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{h^2}{2} + C_2\frac{h^2}{2} + C_3\frac{e^{\lambda h}}{\lambda} + C_4\frac{e^{-\lambda h}}{\lambda}\right)\right)\right.$$

$$\left. +Sr\,Sc\left(\Pr Ec(C_3+C_4) + \frac{\Pr Ec}{\lambda}\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(C_3 - C_4\right)\right)\right\}$$

$$A_4 = -Sr\,Sc\left[\Pr Ec(C_3+C_4) + \frac{\Pr Ec}{\lambda}\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(C_3 - C_4\right)\right]$$

$$a_1 = \frac{\lambda^3}{4\lambda-1}(C_3^2 + C_4^2), a_2 = e^{\lambda h}, a_3 = e^{-\lambda h},\quad a_4 = q_1 + \frac{\lambda^3}{4\lambda-1}(C_3^2 e^{2\lambda h} + C_4^2 e^{-2\lambda h}), a_5 = \lambda e^{\lambda h}, a_6 = \lambda e^{-\lambda h},$$

$$a_7 = \frac{2\lambda^4}{4\lambda-1}(C_3^2 e^{2\lambda h} - C_4^2 e^{-2\lambda h}),\ a_8 = \frac{4\lambda^3}{4\lambda-1}(C_3^2 - C_4^2),\quad a_9 = ha_5 - a_2 + 1, a_{10} = -a_6 h - a_3 + 1, a_{11} = a_7 h - a_4 + a_1,$$

$$a_{12} = C_3B_3, a_{13} = C_3B_4 + C_4B_3,\ a_{14} = C_4B_4,\ a_{15} = \frac{4\lambda^3}{1-4\lambda}, a_{16} = C_3^3,$$

$$a_{17} = C_3C_4^2,\ a_{18} = C_3^2C_4,\ a_{19} = C_4^3, a_{20} = 3C_3^2C_4, a_{21} = 3C_3C_4^2,$$

$$a_{22} = B_2 + C_2B_2 - B_3C_4\lambda^2 - B_4C_3\lambda^2,\ a_{23} = B_3 + C_2B_3 + C_3B_2 - \frac{2\lambda^4}{1-4\lambda}C_3^2C_4$$

$$a_{24} = -B_4 - C_2B_4 - B_2C_4 - \frac{2\lambda^4}{1-4\lambda}C_4^2C_3,\ a_{25} = \frac{2\lambda^4}{1-4\lambda}C_3^2C_2 + C_3B_3\lambda^2 + \frac{2\lambda^4}{1-4\lambda}C_3^2$$

$$a_{26} = -\frac{2\lambda^4}{1-4\lambda}C_4^2C_2 + C_4B_4\lambda^2 - \frac{2\lambda^4}{1-4\lambda}C_4^2,\ a_{27} = \frac{2\lambda^5}{1-4\lambda}C_3^3, a_{28} = \frac{2\lambda^5}{1-4\lambda}C_4^2,\ B_1 = a_1 - a_8,$$

$$B_2 = \frac{1}{h}\left\{a_4 - a_1 + (1-a_2)\left(\frac{1}{a_9}\left[a_{11} - a_{10}\left(\frac{a_8a_9 - a_{11}}{a_9 - a_{10}}\right)\right]\right) + (1-a_3)\left(\frac{a_8a_9 - a_{11}}{a_9 - a_{10}}\right)\right\},$$

$$B_3 = \frac{1}{a_9}\left[a_{11} - a_{10}\left(\frac{a_8a_9 - a_{11}}{a_9 - a_{10}}\right)\right],\ B_4 = \frac{a_8a_9 - a_{11}}{a_9 - a_{10}},$$

$$D_1 = \frac{1}{h}\left\{\Pr Ec\left(a_{13}h^2\lambda^2 + \frac{a_{12}e^{2\lambda h} + a_{14}e^{-2\lambda h}}{2} + a_{15}\left[a_{17}e^{-\lambda h} + a_{18}e^{\lambda h} + \frac{a_{16}e^{3\lambda h} + a_{19}e^{-3\lambda h}}{9}\right]\right.\right.$$

$$\left.\left. +\lambda^4\left(\frac{a_{16}e^{3\lambda h} + a_{19}e^{-3\lambda h}}{9} + a_{20}e^{\lambda h} + a_{21}e^{-\lambda h}\right)\right)\right.$$

$$+2\Pr Ec\frac{M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{a_{22}h^2}{2}+\frac{a_{23}e^{\lambda h}+a_{24}e^{-\lambda h}}{\lambda}+\frac{a_{25}e^{2\lambda h}+a_{26}e^{-2\lambda h}}{4\lambda^2}+\frac{a_{27}e^{3\lambda h}+a_{28}e^{-3\lambda h}}{9\lambda^2}\right)-D_2\Bigg\}$$

$$D_2=\Pr Ec\left\{2\left(\frac{a_{12}+a_{14}}{4}+a_{15}\left[a_{17}+a_{18}+\frac{a_{16}+a_{19}}{9}\right]\right)+\lambda^4\left(a_{20}+a_{21}+\frac{a_{16}+a_{19}}{9}\right)\right\}$$

$$+\Pr Ec\frac{2M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{a_{23}+a_{24}}{\lambda}+\frac{a_{25}+a_{26}}{4\lambda^2}+\frac{a_{27}+a_{28}}{4\lambda^2}\right)$$

$$F_2=-SrSc\left\{\Pr Ec\left\{2\left(\frac{a_{12}+a_{14}}{4}+a_{15}\left[a_{17}+a_{18}+\frac{a_{16}+a_{19}}{9}\right]\right)+\lambda^6\left(a_{20}+a_{21}+\frac{a_{16}+a_{19}}{9}\right)\right\}\right.$$

$$\left.+\Pr Ec\frac{2M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{a_{23}+a_{24}}{\lambda}+\frac{a_{25}+a_{26}}{4\lambda^2}+\frac{a_{27}+a_{28}}{4\lambda^2}\right)\right\}$$

$$F_1=\frac{1}{h}\left\{-SrSc\left[\Pr Ec\left(a_{13}h^2\lambda^2+\frac{a_{12}e^{2\lambda h}+a_{14}e^{-2\lambda h}}{2}+\right.\right.\right.$$

$$2a_{15}\left[a_{17}e^{-\lambda h}+a_{18}e^{\lambda h}+\frac{a_{16}e^{3\lambda h}+a_{19}e^{-3\lambda h}}{9}\right]+\lambda^4\left[\frac{a_{16}e^{3\lambda z}+a_{19}e^{-3\lambda z}}{9}+a_{20}e^{\lambda z}+a_{21}e^{-\lambda z}\right]\Bigg]\Bigg]+$$

$$+\Pr Ec\frac{2M^2\sin^2\alpha}{1+m^2\sin^2\alpha}\left(\frac{a_{22}h^2}{2}+\frac{a_{23}e^{\lambda h}+a_{24}e^{-\lambda h}}{\lambda}+\frac{a_{25}e^{2\lambda h}+a_{26}e^{-2\lambda h}}{4\lambda^2}+\frac{a_{27}e^{3\lambda h}+a_{28}e^{-3\lambda h}}{9\lambda^2}\right)-F_2\Bigg\}$$